TIME:3 HOURS **MARKS:80**

NOTE:

1. *Attempt ANY 3 questions from Q.2 to Q.6*
2. *Figures to right indicate marks.*
3. *Q.1 is COMPULSORY.*
4. *Use of scientific calculators allowed.*

(Q.1)

a) Find the laplaces transform of $te^t \sin 2t \cos 2t$ (05)

b) Find the inverse laplaces transform of $\frac{s+2}{s^2(s+3)}$ (05)

c) Determine wether the function $f(z) = x^2 - y^2 + 2ixy$ is analytic and if so find its derivative (05)

d) Find the Fourier series for $f(x) = e^{-|x|}$ in the interval (-π,π) (05)

(Q.2)

a) Evaluate $\int_0^\infty \frac{e^{-t} - \cos t}{te^{4t}} dt$ (06)

b) Find the Z-transform of $f(k) = \begin{cases} 3^k, & k < 0 \\ 2^k, & k \geq 0 \end{cases}$ (06)

c) Show that the function u=2x(1-y) is a harmonic function. Find oits harmonic conjugate and corresponding analytic function (08)

(Q.3)

a) Find the equation of the line of regression of y on x for the following data (06)

X	10	12	13	16	17	20	25
Y	19	22	24	27	29	33	37

b) Find the bilinear transformation which maps z=2,1,0 onto w=1,0,I (06)

c) Obtain the expansion of f(x)=x(π-x) ,0<x<π as a half range cosine series. Hence show that $\sum_1^\infty \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$ (08)

(Q.4)

a) Find the inverse laplaces by using convolution theorem $\frac{1}{(s^2+1)(s^2+9)}$ (06)

b) Calculate the coefficient of correlation between Price and Demand (06)

Price : 2, 3 ,4 ,7 ,4

Demand : 8, 7, 3, 1, 1

c) Find the inverse Z-transform for the following ; (08)

i) $\frac{z}{z-5}, |z| < 5$ ii) $\frac{1}{(z-1)^2}, |z| > 1$

(Q.5)

a) Find the laplaces transform of $e^{-t} \sin t \; H(t-\pi)$ (06)

b) Show that the set of functions $\{\sin x, \sin 3x, \sin 5x, \ldots\}$ is orthogonal over [0, 2π] .Hence construct orthogpnal set of functions (06)

c) Solve using Laplace transform, $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 3te^{-t}$ (08)

given y(0) = 4 and y '(0) = 2.

(Q.6)

a) Find a complex of Fourier series for f(x)=3x in (0,2π) (06)

b) If f(z) is an analytic function with constant modulus then, prove that f(z) is constant (06)

c) Fit a curve of the form $y=ax^b$ to the following data (08)

X	1	2	3	4
Y	2.5	8	19	50