SE - IT Sem-III choice Based

(3 Hours) (Total Marks : 80)

Note: 1. Question no. **1** is compulsory.
2. Attempt any **three** questions out of remaining **five** questions.

20 NOV 2018

Q.1.[a] Evaluate $L[\sin 2t \cos t \cosh 2t]$. [5]

[b] How many friends must you have to guarantee that atleast five of them have birthday in the same month. [5]

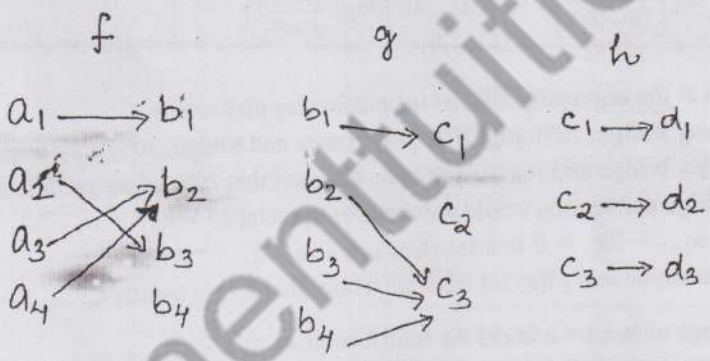
[c] Determine the constants a, b, c, d, e so that the function [5]
$f(z) = ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2 + i\left(4x^3y - exy^3 + 4xy\right)$ is analytic.

[d] Out of one lakh people 51500 are female and 48500 are male. Among the females 9000 are singers, among the males 30200 are singers. A person chosen randomly. If A, B, C are the events that a singer is chosen, a female is chosen and male is chosen respectively then find (i) P(A/B) (ii) P(A/C) (iii) P(A/C) (iv) P(C/A). [5]

Q.2. [a] Using Venn diagram show that $P \cap (Q \oplus R) = (P \cap Q) \oplus (P \cap R)$. [6]

[b] Evaluate $L\{f(t)\}$ where $f(t) = \begin{cases} 1 & 0 \le t < a \\ -1 & a < t < 2a \end{cases}$ and $f(t+2a) = f(t)$. [6]

[c] Let f, g, h be the functions shown in the diagraph : [8]



Find : (i) g∘f, h∘(g∘f), (h∘g)∘f , h^{-1}
(ii) Identify onto and one-one function for 3 of them.

Q.3. [a] Find analytic function $f(z) = u + iv$ where $v = \frac{x}{x^2 + y^2} + \cosh x \cos y$. [6]

[b] Solve $(D^2 + 2D + 5)y = e^{-t} \sin t$, *when* $y(0) = 0,\ y'(0) = 1$. [6]

[c] Evaluate (i) $L\left\{\frac{1}{t}(1 - \cos t)\right\}$ [8]

(ii) $\int_0^\infty e^{-t}\left(\int_0^t u^4 \sinh u \cosh u \, du\right) dt$

Q.4. [a] Evaluate using convolution theorem $L^{-1}\left[\frac{(s+2)}{(s^2+4s+8)^2}\right]$ [6]

[b] Find bilinear transformation which maps the points $z=-1,1,\infty$ onto $w=-i,-1,i$. [6]

[c] Three machines A, B and C produce respectively 25%, 35% and 40% of the total number of items of a factory. The percentages of defective output of these machines are respectively 5%, 4% and 2%. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine A. [8]

Q.5. [a] Suppose repetitions are not permitted.
(i) How many four- digit numbers can be formed from the digits 1, 2, 3, 5, 7, 8? [6]
(ii) How many of the numbers in part (a) are less than 4000?
(iii) How many of the numbers in part (a) are multiples of 5?

[b] Let A={1,2,3,4,12} and let R be the relation on A defined by xRy if and only if "x divides y", Show that (A,R) is a PO set. Draw the diagraph of R. [6]

[c] Evaluate (*i*) $L^{-1}\left[\frac{e^{-5s}}{(s-2)^4}\right]$ (*ii*) $L^{-1}\left[\log\left(\frac{s+3}{s+5}\right)\right]$ [8]

Q.6. [a] It is known that at the university 60% of the professors play tennis, 50% of them play bridge, 70% jog, 20% play tennis and bridge, 30% play tennis and jog, 40% play bridge and jog. If someone claimed that 20% of the professors jog and play bridge and tennis, would you believe this claim? Why? [6]

[b] Solve $a_{r+2} + 2\,a_{r-1} - 3a_r = 0$ that satisfies $a_0 = 1$, $a_1 = 2$. [6]

[c] (i) If f(z) is an analytic and | f(z) | is constant, show that f(z) is constant. [8]

(ii) Find the image of |z-ai| = a under the transformation $w=\frac{1}{z}$.

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