



Discrete Mathematics

JUN 18

Computer Engineering (Semester 3)

Total marks: 80

Total time: 3 Hours

INSTRUCTIONS

- (1) Question 1 is compulsory.
- (2) Attempt any **three** from the remaining questions.
- (3) Draw neat diagrams wherever necessary.

1.a. Prove by induction that the sum of the cubes of three consecutive numbers is divisible by 9. (5 marks)

1.b. Find the generating function for the following finite sequences.

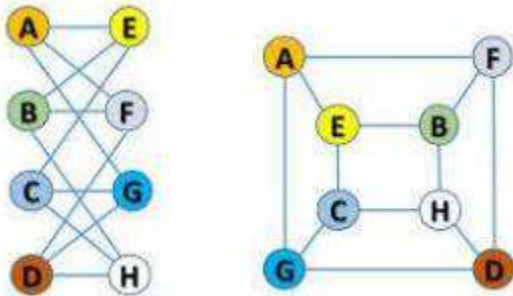
i) 2,2,2,2,2

ii) 1,1,1,1,1,1 (5 marks)

1.c. A box contains 6 white balls and 5 red balls. In how many ways 4 balls can be drawn from the box if, i) they are to be any color ii) all the balls to be of the same color. (5 marks)

1.d. Find the complement of each element in D_{30} . (5 marks)

2.a. Define isomorphism of graphs. Find if the following two graphs are isomorphic. If yes, find the one-to-one correspondence between the vertices.



(8 marks)

2.b. In a certain college 4% of the boys and 1% of the girls are taller than 1.8 mts. Furthermore 60% of the students are girls. If a student selected at random is taller than 1.8 mts. what is the probability that the student was a boy? Justify your answer. (8 marks)



2.c. Prove $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. (8 marks)

3.a. Prove that set $G = \{1, 2, 3, 4, 5, 6\}$ is a finite abelian group of order 6 with respect to multiplication module 7. (8 marks)

3.b. Let $A = \{1, 2, 3, 4, 5\}$, let $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5)\}$ and $S = \{(1, 1), (2, 2), (3, 3), (4, 4), (4, 5), (5, 4), (4, 5)\}$ be the relations on A. Find the smallest containing the relation R and S. (8 marks)

3.c. Test whether the following is one-to-one, onto or both.
 $f : \mathbb{Z} \Rightarrow \mathbb{Z}, f(x) = x^2 + x + 1$ (4 marks)

4.a. Show that the (2,5) encoding function $e: B^2 \Rightarrow B^5$ defined by $e(00) = 00000$ $e(01) = 01110$

$e(10) = 10101$ $e(11) = 11011$ is a group of code.

How many errors will it detect and correct? (8 marks)

4.b. Let $H =$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Be a parity check matrix. Determine the group code. $eH: B^3 \Rightarrow B^6$ (8 marks)

4.c. How many friends must you have to guarantee at least five of them will have birthdays in the same month? (4 marks)

5.a Let G be a set of rational numbers other than 1. Let $*$ be an operation on G defined by $ab = a + b - ab$ for all $a, b \in G$. prove that $(G, *)$ is a group. (8 marks)

5.b. Solve $ar - 7ar - 1 + 10ar - 2 = 6 + 8r$ given $a_0 = 1, a_1 = 2$ (8 marks)



5.c. Let $A = \{a, b, c, d, e, f, g, h\}$. Consider the following subsets of A ,

$A_1 = \{a, b, c, d\}$ $A_2 = \{a, c, e, g, h\}$

$A_3 = \{a, c, e, g\}$ $A_4 = \{b, d\}$ $A_5 = \{f, h\}$

Determine whether the following is a partition of A or not. Justify your answer.

i) $\{A_1, A_2\}$ ii) $\{A_3, A_4, A_5\}$

(4 marks)

6.a. Draw the Hasse Diagram of the following sets under order relation divides and indicate which are chains. Justify your answers.

i) $A = \{2, 4, 12, 24\}$

ii) $A = \{1, 3, 5, 15, 30\}$

(8 marks)

6.b. Let the functions f, g and h defined as follows:

$f: \mathbb{R} \Rightarrow \mathbb{R}, f(x) = 2x + 3$ $f: \mathbb{R} \Rightarrow \mathbb{R}, f(x) = 2x + 3$

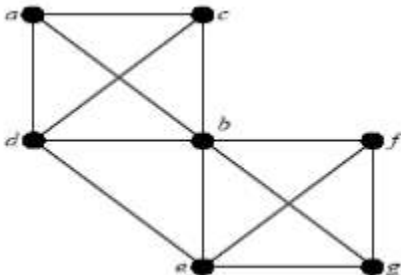
$g: \mathbb{R} \Rightarrow \mathbb{R}, g(x) = 3x + 4$ $g: \mathbb{R} \Rightarrow \mathbb{R}, g(x) = 3x + 4$

$h: \mathbb{R} \Rightarrow \mathbb{R}, h(x) = 4x$ $h: \mathbb{R} \Rightarrow \mathbb{R}, h(x) = 4x$

Find $g \circ f$, $f \circ g$, $g \circ f \circ h$

(8 marks)

6.c. Determine Euler cycle and path in graph shown below



(4 marks)