Discrete Mathematics

DEC 17

Computer Engineering (Semester 3)

Total marks: 80
Total time: 3 Hours

INSTRUCTIONS
(1) Question 1 is compulsory.
*(2) Attempt any **three** from the remaining questions.*
(3) Draw neat diagrams wherever necessary.

1.a. Prove that 1.1! +2.2! + 3.3! + ... + n.n! = (n+1)!- 1,
where n is a positive integer.   (5 marks)
1.b. Let A = {a,b,c} . Show that (P(A), $\subseteq\subseteq$) is a poset and
draw its Hasse diagram .   (5 marks)
1.c. Explain the terms-
i) Lattice

ii) Poset

iii) Normal subgroup

iv) Group

v) Plannar Graph   (5 marks)

1.d. Comment whether the function f is one to one or onto.
consider the function: f: N $\Rightarrow\Rightarrow$N is a set of natural numbers including zero.
f(j)=j2+2f(j)=j2+2   (5 marks)

2.a. Find the number of ways a person can distribute Rs.601 as pocket
money to his three sons, so that no son should receive more than the
combined total of the other two.
(Assume no fraction of a rupee is allowed)   (6marks)

2.b. Let A = {a1, a2, a3, a4, a5} and let R be a relation on A whose matrix is

$$M_R = \begin{vmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{vmatrix}$$

Find MRMR* by Warshall's algorithm   (6 marks)

2.c. Find the complete solution of the recurrence relation: an+2an−1=n+3an+2an−1=n+3 for n≥1n≥1 and with a0=3a0=3   (4 marks)

2.d. let f : R ⇒⇒ R defined as f(x)=x3f(x)=x3and

let g : R ⇒⇒ R defined as g(x)=4x2+1g(x)=4x2+1

Find out g0f,f0g,f2,g2g0f,f0g,f2,g2   (4 marks)

3.a. Given that a student had prepared, the probability of passing a certain entrance exam is 0.99 . Given that a student did not prepare, the probability of passing the entrance exam is 0.05. Assume that the probability of preparing is 0.7 . The student fails in the exam. What is the probability that he or she did not prepare?   (6 marks)

3.b. Define equivalence relation with example. let 'T' be a set of triangles in a plane and define R as the R = {(a, b) a,b ∈∈ T and a is congruent to b } then show that R is an equivalence relation.   (4 marks)

3.c. Let A=B=R , the set of real numbers

let f: A ⇒⇒ B, be given by the formula f(x) = 2x3−12x3−1 and let g : B ⇒⇒ A be given by

$$g(y) = \sqrt[3]{\tfrac{1}{2}y + \tfrac{1}{2}}$$

Show that f is a bijection between A and B and

g is a bijection between B and A.   (4 marks)

3.d. Let ZnZn denote the set of integers {0,1,2.......n-1}. Let O be binary operation on znzn such that a O b = the remainder of ab divided by n.

i) Consruct the table for the operation O for n=4.

ii) Show that (ZnZn O)is a semigroup for any n.   (4 marks)

4.a. i) Among 50 students in a class, 26 got an A in the first examination
and 21 got an A in the second examination. If 17 students did not get
an A in either examination , how many students got an A in both examinations?
ii) If the number of students who got an A in the first examination is equal

to that in the second examination, if the total numbers of students who

got an A in exactly one examination is 40 and if 4 students did not get an

A in either examination, then determine the number of students who got

an A in the first examination only, who got an A in the second examination

only; and who got an A in both the examination.   (6 marks)

4.b. Consider the (2,5) group encoding function
$e : B2 \Rightarrow B5B2 \Rightarrow B5$ defined by
e(00) = 00000 e(01) = 01110

e(10) = 10101 e(11) = 11011

Decode the following works relative to a maximum likelihoods

decoding function.

i) 11110 ii) 10011 iii)10100   (4 marks)

4.c. i) Is every Eulerian graph a Hamiltonian?
ii) Is every Hamiltonian graph a Eulerian?

Explain with the necessary graph.   (4 marks)

4.d. Given the parity check matrix

$$H = \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{vmatrix}$$

Find the minimum distance of the code generated by H.
How many errors it can detect and correct?   (4 marks)

5.a. Explain the Pigeonhole principle and Extended Pigeonhole principle.
Show that in any room of people who have been doing some handshaking
there will always be atleast two people who have shaken hands the same
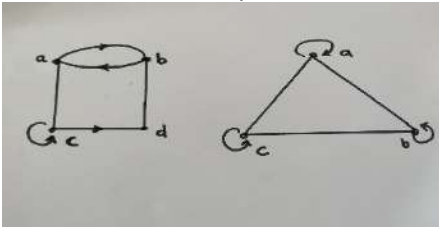number of times.   (6 marks)

5.b. Determine whether the Poset with the following Hasse diagrams are lattices or not. Justify your answer.
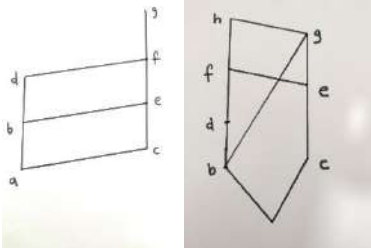
(6 marks)

5.c. From the following digraphs, write the relation a set of ordered pairs. Are the relations equivalence relations?

(4 marks)

5.d. For the set X = {2,23,6,12,24,36} , is a relation ≤≤ is defined as x ≤≤ y if x divides y, Draw the Hasse diagram for (X, ≤≤) . Answer the following:
i) What are the maximal and minimal elements?

ii) give one example of a chain and antichain?

iii) Is the Poset a lattice? (4 marks)

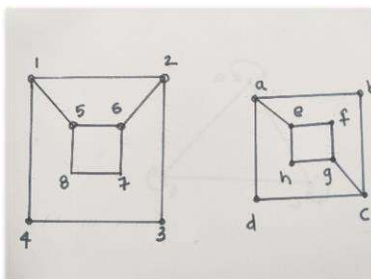**6.a.** Prove that the set {1,2,3,4,5,6} is group under multiplication modulo 7. (6 marks)
6.b. Given a generating function, find out corresponding sequence.
i) $13-6x13-6x$
ii) $x1-5x+6x2x1-5x+6x2$ (6 marks)
6.c. Determine whether following graphs are isomorphic or not.

(4 marks)

6.c. Prove the following (use laws of set theory)
A x (X ∩∩ Y) = (A x X) ∩∩ (A x Y) (4 marks)