



APPLIED MATHS III

DEC-2017

S.E.SEM-III

Total marks: 80

Total time: 3 Hours

INSTRUCTIONS:

- (1) Question 1 is compulsory.
- (2) Attempt any three from the remaining questions.
- (3) Draw neat diagrams wherever necessary.

Q.1) (a) Find the Laplace transform of $\sin^2 3t$ (20)

(b) Prove that $f(z) = \log z$ is analytic

(c) Obtain Fourier series for $f(x) = x^2$ in $(-2, 2)$

(d) Find the Z-transform of $\cos 2k, k \geq 0$

Q.2) (a) Prove that $\vec{F} = 2xyz^3\mathbf{i} + x^2z^3\mathbf{j} + 3x^2yz^2\mathbf{k}$ is irrotational. Find the scalar potential of $\vec{F}$ (06)

(b) Find the inverse Laplace Transform using convolution theorem (06)

$$\frac{1}{(s^2 + 6s + 18)^2}$$

(c) Find the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $(0, 2\pi)$ (08)

Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \dots$

Q.3) (a) Find the analytic function $f(z) = u + iv$ if $u + v = \cos x \cosh y - \sin x \sinh y$ (06)

(b) Find inverse transform of $\frac{2z^2 - 10z + 13}{(z-3)^2(z-2)}$, $2 < |z| < 3$ (06)

(c) Solve the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dt} + y = 3te^{-1}$, $y(0) = 4, y'(0) = 2$ using Laplace transform (08)



Q.4)(a) Find the orthogonal trajectory of $x^2 + y^2 - 3xy + 2y = c$ (06)

(b) Using Green's theorem evaluate $\int_C (x^2 - y) dx + (2y^2 + x) dy$, C is closed path formed by $y=4$, $y=x^2$ (06)

(c) Express the function $f(x) = \begin{cases} \sin x & ; 0 < x \leq \pi \\ 0 & ; x > \pi \end{cases}$ as Fourier's integral. Hence evaluate $\int_0^\infty \frac{\cos(\pi\lambda/2)}{1-\lambda^2} d\lambda$ (08)

Q.5) (a) Find inverse Laplace transform of $\frac{2s^2 - 6s + 5}{s^3 + 6s^2 + 11s - 6}$ (06)

(b) Find the Bilinear Transformation that maps the point $z=1, i, -1$ into $w=1, 0, -1$ (06)

(c) Evaluate using Stokes theorem $\int_C \vec{F} \cdot d\vec{r}$ where C is the boundary of circle $x^2 + y^2 + z^2 = 1, z=0$ and $\vec{F} = yz\mathbf{i} + zx\mathbf{j} + xy\mathbf{k}$ (08)

Q.6)(a) Find the directional derivative of $\phi = x^2 + y^2 + z^2$ in the direction of the line (06)

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{5} \text{ at } (1, 2, 3)$$

(b) Find the complex form of Fourier series for e^{ax} ; $(-\pi, \pi)$ (06)

(c) Find half-range sine series for $f(x) = x(2-x)$ $0 < x < 2$ (08)

$$\text{Hence deduce that } \sum \left(\frac{1}{n^2} \right) = \frac{\pi^6}{945}$$