LAST MOMENT TUITIONS

APPLIED MATHS III

DEC-2018

S.E.SEM-III

Total marks: 80

Total time: 3 Hours

INSTRUCTIONS:

(1) Question 1 is compulsory.

(2) Attempt any three from the remaining questions.

(3) Draw neat diagrams wherever necessary.

Q.1)(a)Find the Laplace transform of $e^{-t}t\cosh 2t$ **(05)**

(b)Find the half range cosine series for f(x)=$\begin{cases} 1\ ,0 < x < \frac{a}{2} \\ -1\ ,\frac{a}{2} < x < a \end{cases}$ **(05)**

(c)Find $\nabla\left(\bar{a}.\nabla\frac{1}{r}\right)$ $where\bar{a}$ is a constant vector. **(05)**

(d)Show that the function $f(z)=z^3$ is analytic and find f'(z) in terms of z **(05)**

Q.2)(a)Find the inverse Z-transform of $F(z)=\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}$,3<z<4. **(06)**

(b)Find the analuytic function whose imaginary part is $\tan^{-1}\left(\frac{y}{x}\right)$ **(06)**

(c)Obtain Fourier series for the function f(x)=$\begin{cases} \frac{\pi}{2}+x, -\pi < x < 0 \\ \frac{\pi}{2}-x, 0 < x < \pi \end{cases}$ **(08)**

Hence,deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$and $\frac{\pi^4}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots$

Q3)(a)Find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$ using covolution theorem **(06)**

(b)Show that the set of functions$\emptyset_n(x) = \sin\left(\frac{n\pi x}{l}\right)$, n=1,2,3..is orthogonal in [0,l] **(06)**

(c)Using Greens theorem evaluate $\emptyset_c\left(e^{x^2} - xy\right)dx - (y^2 - ax)dy$ where C is the circle $x^2+y^2=a^2$ **(08)**

Q.4)(a)Find Laplace transform of f(t)=$\begin{cases} \frac{t}{a}, 0 < t \leq a \\ \frac{(2a-t)}{a}, a < t < 2a \end{cases}$ and f(t)=f(t+2a). **(06)**

(b)Prove that a vector field $\bar{f}$ is irrotational and hence find its scalar potential **(06)**

$\bar{f}$=(ysinz-sinx)i+(xy cosz+y^2)k.

(c)Obtain the fourier expansion of f(x)=$\left(\frac{-x}{2}\right)^2$ in the interval 0≤x≤ 2 and f(x+2)=f(x).Also deduce that **(08)**

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} +..$$

Q.5)(a)Use Gauss's divergence theorem to evaluate $\iint_s \bar{N}.\overline{Fds}$ where $\bar{F} = 4xi + 3yj - 2zk$ and S is **(06)**

bounded by x=0,y=0,z=0 and 2x+2y+z=4.

(b)Find the Z-transform of f(k)=ke-ak,k≥ 0. **(06)**

(c) i.Find $L^{-1}\left[\frac{s+2}{s^2(s+3)}\right]$. **(08)**

ii.Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$.

Q.6)(a)Solve using Laplace's transform **(06)**

(D2+3D+2)y=2(t2+t+1), with y(0)=2 and y'(0)=0.

(b)Find the bilinear transformation which makes the points Z=1 ,I,-1 onto the points W=I,0,-I **(06)**

(c)Find Fourier sine integral of f(x)=$\begin{cases} x, 0 < x < 1 \\ 2 - x, 1 < x < 2 \\ 0, x > 2 \end{cases}$ **(08)**